Tesseroids Documentation

Release 1.1

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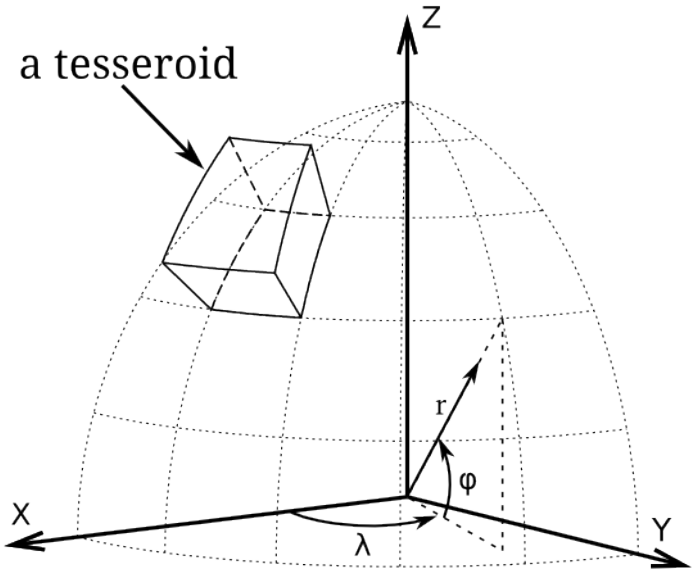
CONTENTS

Forward modeling of gravitational fields in spherical coordinates

Tesseroids is a collection of **command-line programs** for modeling the gravitational potential, acceleration, and gradient tensor.

For more information visit the official site (http://leouieda.github.com/tesseroids).

The geometric element used in the modeling processes is a **spherical prism**, also called a **tesseroid**. *Tesseroids* also contains programs for modeling using **right rectangular prisms**, both in **Cartesian** and **spherical coordinates**.



Tesseroids is developed by Leonardo Uieda (http://fatiando.org/people/uieda/) in cooperation with Carla Braitenberg (http://www2.units.it/geodin/biobraitenberg.html).

If you use *Tesseroids* in your research, please consider **citing** it in your publications. See *Citation* for more information.

As of version 1.1, *Tesseroids* is available under the **BSD license**. This means that it can be reused and remixed with fewer restrictions. See the *license text* for more information.

The **source code** of *Tesseroids* is hosted on GitHub (https://github.com/leouieda/tesseroids). There you can browse the code, create your own fork, and start contributing! Get in touch (http://fatiando.org/people/uieda) to see how you can help.

This **documentation** explains how to *install* and *use* Tesseroids. It also contains some *theoretical background* to get you up-to-date with the terms and equations that we use. The *cookbook* has a few example recipes and the expected output (I recommend starting here if you just want a quick peek).

CHAPTER

ONE

TESSEROIDS LICENSE

CHAPTER

TWO

CITATION

To cite *Tesseroids* in publications, please use:

Uieda, L. (2013), Tesseroids: Forward modeling of gravitational fields in spherical coordinates, figshare, http://dx.doi.org/10.6084/m9.figshare.786514, doi:10.6084/m9.figshare.786514.

If you require a more traditional publication (though the **above** citation is prefered), use:

Uieda, L., E. P. Bomfim, C. Braitenberg, and E. Molina (2011), Optimal forward calculation method of the Marussi tensor due to a geologic structure at GOCE height, Proceedings of the 4th International GOCE User Workshop. [pdf (http://fatiando.org/papers/Uieda,etal._2011.pdf)]

If you're a BibTeX user:

```
@article{,
    author = {Uieda, Leonardo},
    title = {Tesseroids: Forward modeling of gravitational fields in
             spherical coordinates},
    journal = {figshare},
    year = {2013},
    doi = {10.6084/m9.figshare.786514},
    url = {http://dx.doi.org/10.6084/m9.figshare.786514}
}

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    title = {Optimal forward calculation method of the Marussi tensor
             due to a geologic structure at GOCE height},
    booktitle = {Proceedings of the 4th International GOCE User
                 Workshop},
    year = {2011},
    url = {http://fatiando.org/papers/Uieda,etal._2011.pdf}
}
```

CHAPTER

THREE

CHANGELOG

3.1 Changes in version 1.1

- the tesseroids license was changed from the GNU GPL to the more permissive BSD license (see *the license text*).
- tess2prism has a new flag –flatten to make the prism model by flattening the tesseroids (i.e., 1 degree = 111km) into Cartesian coordinates (so that they can be used with the prismg* programs).
- tessg* programs have a new flag -t used to control the distance-size ratio for the automatic recursive division of tesseroids.
- **NEW PROGRAMS** prismpots, prismgs, and prismggts, to calculate the prism effects in spherical coordinates. These programs are compatible with the output of tess2prism (see *this recipe* for an example).
- **NEW PROGRAM** tesslayers to generate a tesseroid model of a stack of layers from grids of the thickness and density of each layer. tesslayers complements the functionality of tessmodgen and can be used to generate crustal models, sedimentary basin models, etc. (see *this recipe* for an example).
- tesseroids now strictly follows the ANSI C standard.
- Bug fix: prismpot, prismgx, prismgy, prismgz, and prismgxy had problems with a log(z + r) when the computation point was bellow the top of the prism (zp > prism.z1). Fixed by calculating on top of the prism when this happens, then changing the sign of the result when needed (only for gz).
- Bug fix: the tessg and prismg family of programs was crashing when the model file is empty. Now they fail with an error message.

3.2 Changes in version 1.0

Tesseroids 1.0 was completely re-coded in the C programming language and is much faster and more stable than the 0.3 release. Here is a list of new features:

- tesspot and tessg* programs now take the computation points as input, allowing for custom grids.
- tesspot and tessg* programs now automatically subdivide a tesseroid if needed to maintain GLQ precision (this makes computations up to 5x faster and safer).
- Automated model generation using program tessmodgen.
- Regular grid generation with program tessgrd.
- Total mass calculation with program tessmass.
- Programs to calculate the gravitational fields of right rectangular prisms in Cartesian coordinates.
- HTML User Manual and API Reference generated with Doxygen.
- Easy source code compilation with SCons.

CHAPTER

FOUR

INSTALLATION INSTRUCTIONS

4.1 Downloading

You can download the source and binary distributions from the Google Code project site (http://code.google.com/p/tesseroids/).

4.2 Pre-compiled binaries

If you downloaded a pre-compiled binary distribution, simply unpack in the desired directory.

The executables will be in the `bin` folder, the pdf documentation in the `doc` folder, and example scripts in the `cookbook` folder.

4.3 Compiling from source

To build Tesseroids you'll need:

- A C compiler (like GCC (http://gcc.gnu.org))
- SCons (http://www.scons.org/)

4.3.1 Setting up SCons

Tesseroids uses the build tool SCons (http://www.scons.org/). A `SConstruct` file (`Makefile` equivalent) is used to define the compilation rules. You will have to download and install SCons in order to easily compile Tesseroids. SCons is available for both GNU/Linux and Windows. Building should work the same on both platforms.

SCons requires that you have Python (http://www.python.org) installed. Check the SCons website (http://www.scons.org/) for more information. Python is usually installed by default on most GNU/Linux systems.

Under Windows you will have to put SCons on your `PATH` environment variable in order to use it from the command line. It is usually located in the `Scripts` directory of your Python installation.

On GNU/Linux, SCons will generally use the GCC (http://gcc.gnu.org) compiler to compile sources. On Windows it will search for an existing compiler. We recomment that you install GCC on Windows using MinGW (http://mingw.org/).

4.3.2 Compiling

First, download a source distribution. Unpack the archive anywhere you want (e.g., `~/tesseroids` or `C:\tesseroids` or whatever). To compile, go to the directory where you unpacked (e.g., `~/tesseroids` etc.) and type in a terminal (or `cmd.exe` on Windows):

```
scons
```

The executables will be placed on a `bin` folder.

To clean up the build, run:

```
scons -c
```

This will delete all object files and executables.

4.3.3 Testing the build

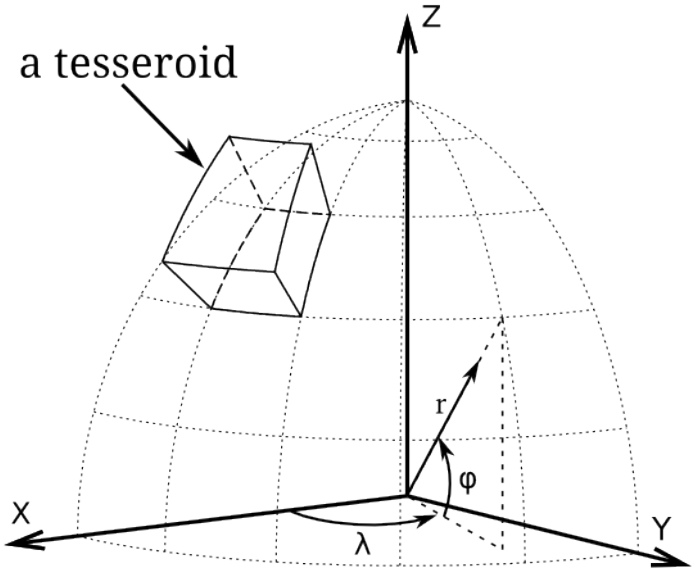
After the compilation, a program called `tesstest` will be placed in the directory where you unpacked the source. This program runs all the unit tests (https://en.wikipedia.org/wiki/Unit_testing) in the `test` directory. If all tests pass, the compilation probably went well. If any test fails, please submit a bug (http://code.google.com/p/tesseroids/issues/list) with the output of `tesstest`.

CHAPTER

FIVE

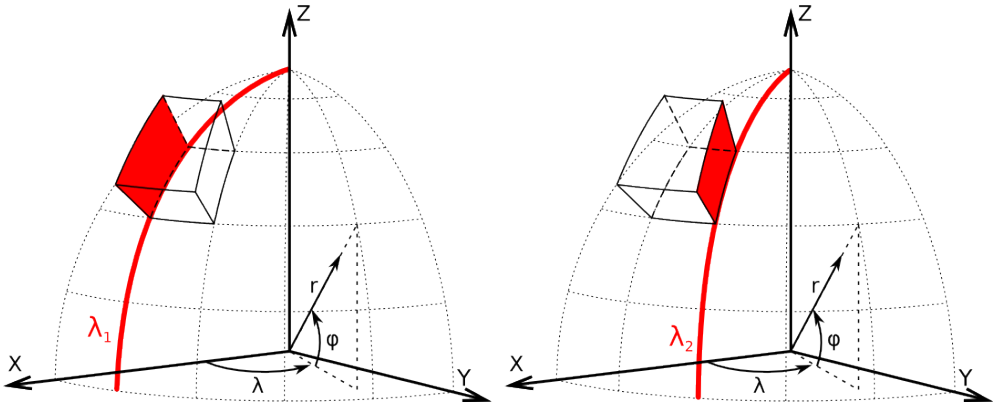
THEORERICAL BACKGROUND

5.1 What is a tesseroid anyway?

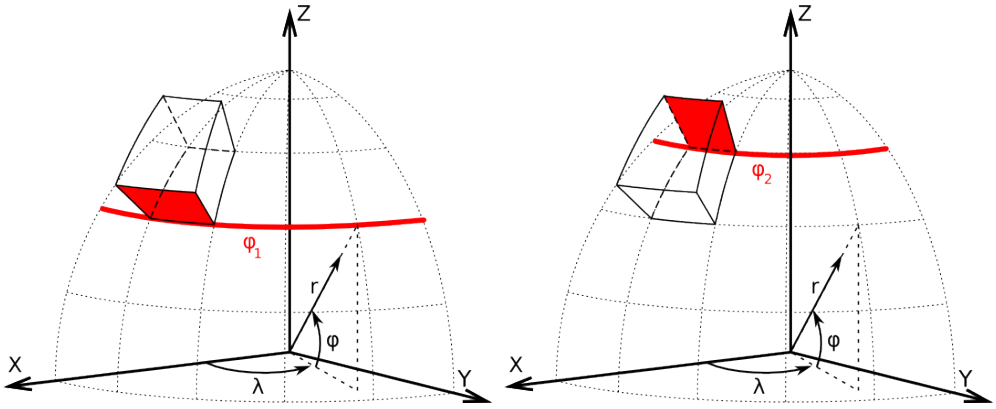


A tesseroid, or spherical prism, is segment of a sphere. It is delimited by:

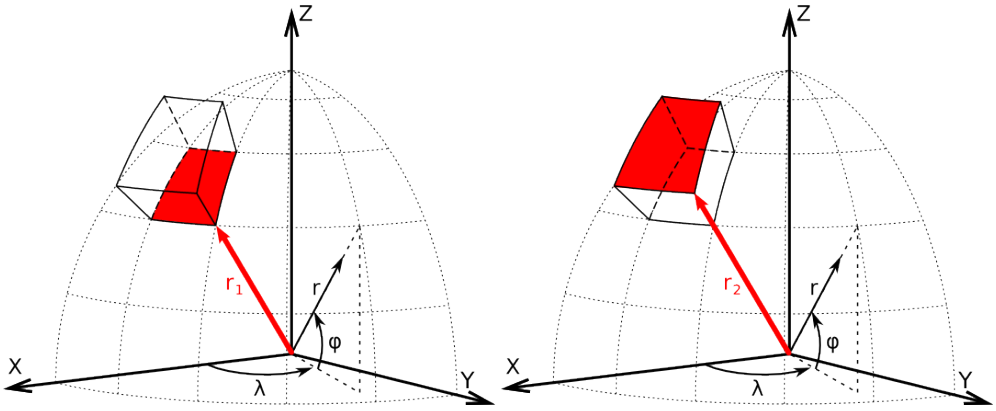
1. 2 meridians, λ_1 and λ_2



2. 2 parallels, ϕ_1 and ϕ_2



3. 2 spheres of radii r_1 and r_2



5.2 About coordinate systems

The figure bellow shows a tesseroid, the global coordinate system (X, Y, Z), and the local coordinate system (x, y, z) of a point P.

The global system has origin on the center of the Earth and Z axis aligned with the Earth's mean rotation axis. The X and Y axis are contained on the equatorial parallel with X intercepting the mean Greenwich meridian and Y completing a right-handed system.

The local system has origin on the computation point P. It's z axis is oriented along the radial direction and points away from the center of the Earth. The x and y axis are contained on a plane normal to the z axis. x points North and y East.

The gravitational attraction and gravity gradient tensor of a tesseroid are calculated with respect to the local coordinate system of the computation point P.

Warning: The g_z component is an exception to this. In order to conform with the regular convention of z-axis pointing toward the center of the Earth, this component **ONLY** is calculated with an inverted z axis. This way, gravity anomalies of tesseroids with positive density are positive, not negative.

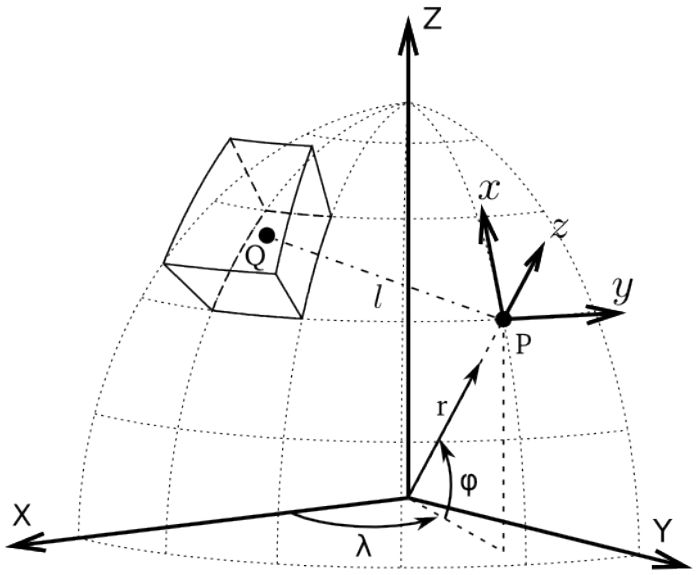


Figure 5.1: View of a tesseroid, the integration point Q, the global coordinate system (X, Y, Z), the computation P and it's local coordinate system ($x,\ y,\ z$). $r,\ \phi,\ \lambda$ are the radius, latitude, and longitude, respectively, of point P.

5.3 Gravitational fields of a tesseroid

The gravitational potential of a tesseroid can be calculated using the formula

$$V(r, \phi, \lambda) = G\rho \int_{\lambda_1}^{\lambda_2} \int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} \frac{1}{\ell} \kappa \, dr' d\phi' d\lambda'$$

The gravitational attraction can be calculated using the formula (Grombein et al., 2013):

$$g_\alpha(r, \phi, \lambda) = G\rho \int_{\lambda_1}^{\lambda_2} \int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} \frac{\Delta_\alpha}{\ell^3} \kappa \, dr' d\phi' d\lambda' \quad \alpha \in \{x, y, z\}$$

The gravity gradients can be calculated using the general formula (Grombein et al., 2013):

$$g_{\alpha\beta}(r, \phi, \lambda) = G\rho \int_{\lambda_1}^{\lambda_2} \int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} I_{\alpha\beta}(r', \phi', \lambda') \, dr' d\phi' d\lambda' \quad \alpha, \beta \in \{x, y, z\}$$

$$I_{\alpha\beta}(r', \phi', \lambda') = \left(\frac{3\Delta_\alpha \Delta_\beta}{\ell^5} - \frac{\delta_{\alpha\beta}}{\ell^3} \right) \kappa \quad \alpha, \beta \in \{x, y, z\}$$

where ρ is density, $\{x, y, z\}$ correspond to the local coordinate system of the computation point P (see *the tesseroid figure*), $\delta_{\alpha\beta}$ is the Kronecker delta

(http://en.wikipedia.org/wiki/Kronecker_delta), and

$$\begin{aligned}
\Delta_x &= r' K_\phi \\
\Delta_y &= r' \cos\phi' \sin(\lambda' - \lambda) \\
\Delta_z &= r' \cos\psi - r \\
\ell &= \sqrt{r'^2 + r^2 - 2r'r\cos\psi} \\
\cos\psi &= \sin\phi \sin\phi' + \cos\phi \cos\phi' \cos(\lambda' - \lambda) \\
K_\phi &= \cos\phi \sin\phi' - \sin\phi \cos\phi' \cos(\lambda' - \lambda) \\
\kappa &= r'^2 \cos\phi'
\end{aligned}$$

ϕ is latitude, λ is longitude, and r is radius.

Note: The **gravitational attraction** and **gravity gradient tensor** are calculated with respect to (x, y, z), the **local coordinate system** of the computation point P.

5.4 Numerical integration

The above integrals are solved using the Gauss-Legendre Quadrature rule (Asgharzadeh et al., 2007):

$$g_{\alpha\beta}(r, \phi, \lambda) \approx G\rho \frac{(\lambda_2 - \lambda_1)(\phi_2 - \phi_1)(r_2 - r_1)}{8} \sum_{k=1}^{N^\lambda} \sum_{j=1}^{N^\phi} \sum_{i=1}^{N^r} W_i^r W_j^\phi W_k^\lambda I_{\alpha\beta}(r'_i, \phi'_j, \lambda'_k) \; \alpha, \beta \in \{1, 2, 3\}$$

where W_i^r, W_j^ϕ, and W_k^λ are weighting coefficients and N^r, N^ϕ, and N^λ are the number of quadrature nodes (i.e., the order of the quadrature), for the radius, latitude, and longitude, respectively.

5.5 Gravitational fields of a prism in spherical coordinates

The gravitational potential and its first and second derivatives for the right rectangular prism can be calculated in Cartesian coordinates using the formula of Nagy et al. (2000).

However, several transformations have to made in order to calculate the fields of a prism in a global coordinate system using spherical coordinates (see *this figure*).

The formula of Nagy et al. (2000) require that the computation point be given in the Cartesian coordinates of the prism (x^*, y^*, z^* in *this figure*). Therefore, we must first transform the spherical coordinates (r, ϕ, λ) of the computation point P into x^*, y^*, z^*. This means that we must convert vector $\bar{e}$ (from *this other figure*) to the coordinate system of the prism. We must first obtain vector $\bar{e}$ in the global Cartesian coordinates (X, Y, Z):

$$\bar{e}^g = \bar{E} - \bar{E}^*$$

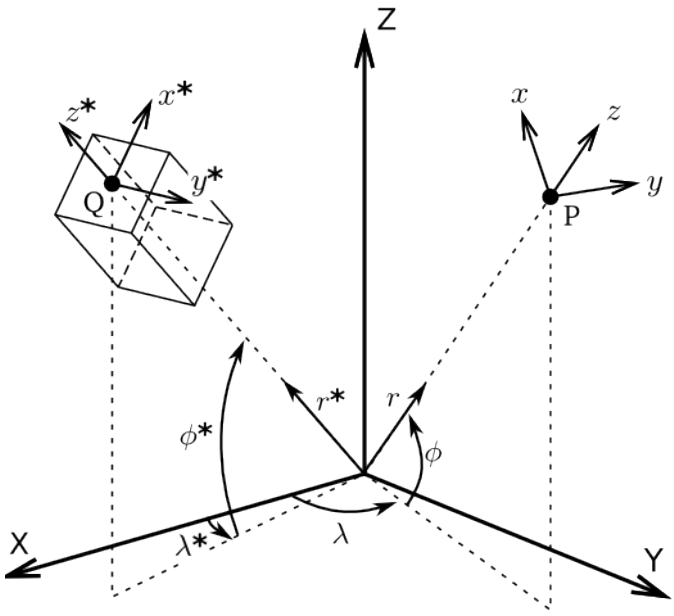


Figure 5.2: View of a right rectangular prism with its corresponding local coordinate system ($x^*,\ y^*,\ z^*$), the global coordinate system (X, Y, Z), the computation P and it's local coordinate system ($x,\ y,\ z$). $r,\ \phi,\ \lambda$ are the radius, latitude, and longitude, respectively.

where $\bar{e}^g$ is the vector $\bar{e}$ in the global Cartesian coordinates and

$$\bar{E} = \begin{bmatrix} r\cos\phi\cos\lambda \\ r\cos\phi\sin\lambda \\ r\sin\phi \end{bmatrix}$$

$$\bar{E}^* = \begin{bmatrix} r^*\cos\phi^*\cos\lambda^* \\ r^*\cos\phi^*\sin\lambda^* \\ r^*\sin\phi^* \end{bmatrix}$$

Next, we transform $\bar{e}^g$ to the local Cartesian system of the prism by

$$\bar{e} = \underbrace{\bar{\bar{P}}_y \bar{\bar{R}}_y(90^\circ - \phi^*)\bar{\bar{R}}_z(180^\circ - \lambda^*)}_{\bar{\bar{W}}}\,\bar{e}^g$$

where $\bar{\bar{P}}_y$ is a deflection matrix of the y axis, $\bar{\bar{R}}_y$ and $\bar{\bar{R}}_z$ are counterclockwise rotation matrices around the y and z axis, respectively (see Wolfram MathWorld (http://mathworld.wolfram.com/RotationMatrix.html)).

$$\bar{\bar{P}}_y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{R}}_y(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

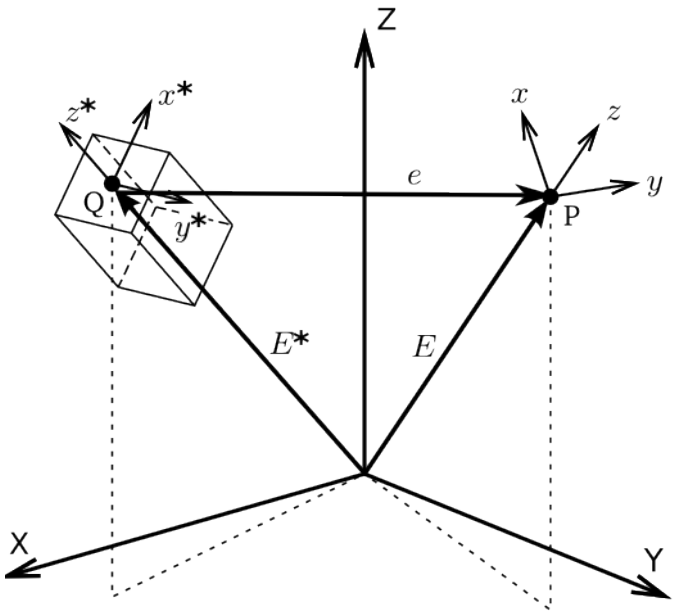


Figure 5.3: The position vectors involved in the coordinate transformations. $\bar{E}^*$ is the position vector of point Q in the global coordinate system, $\bar{E}$ is the position vector of point P in the global coordinate system, and $\bar{e}$ is the position vector of point P in the local coordinate system of the prism ($x^*,\ y^*,\ z^*$).

$$\bar{\bar{R}}_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{W}} = \begin{bmatrix} \cos(90^\circ - \phi^*)\cos(180^\circ - \lambda^*) & -\cos(90^\circ - \phi^*)\sin(180^\circ - \lambda^*) & \sin(90^\circ - \phi^*) \\ -\sin(180^\circ - \lambda^*) & -\cos(180^\circ - \lambda^*) & 0 \\ -\sin(90^\circ - \phi^*)\cos(180^\circ - \lambda^*) & \sin(90^\circ - \phi^*)\sin(180^\circ - \lambda^*) & \cos(90^\circ - \phi^*) \end{bmatrix}$$

Which gives us

$$\bar{e} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Note: Nagy et al. (2000) use the z axis pointing down, so we still need to invert the sign of z.

Vector $\bar{e}$ can then be used with the Nagy et al. (2000) formula. These formula give us the gravitational attraction and the gravity gradient tensor calculated with respect to the coordinate system of the prism (i.e., $x^*,\ y^*,\ z^*$). However, we need them in the coordinate system of the observation point P, where they are measured by GOCE (http://www.esa.int/esaLP/ESAYEK1VMOC_LPgoce_0.html) and calculated for the tesseroids. We perform these transformations via the global Cartesian system (tip: the rotation matrices are orthogonal). $\bar{g}^*$ is the gravity vector in the coordinate system of the prism, $\bar{g}^g$

is the gravity vector in the global coordinate system, and $\bar{g}$ is the gravity vector in the coordinate system of computation point P.

$$\bar{g}^g = \bar{\bar{R}}_z(\lambda^* - 180°)\bar{\bar{R}}_y(\phi^* - 90°)\bar{\bar{P}}_y\bar{g}^*$$

$$\bar{g} = \bar{\bar{P}}_y\bar{\bar{R}}_y(90° - \phi)\bar{\bar{R}}_z(180° - \lambda)\bar{g}^g$$

$$\bar{g} = \bar{\bar{P}}_y\bar{\bar{R}}_y(90° - \phi)\underbrace{\bar{\bar{R}}_z(180° - \lambda)\bar{\bar{R}}_z(\lambda^* - 180°)}_{\bar{\bar{R}}_z(\lambda^* - \lambda)}\bar{\bar{R}}_y(\phi^* - 90°)\bar{\bar{P}}_y\bar{g}^*$$

$$\bar{g} = \bar{\bar{R}}\bar{g}^*$$

$$\bar{\bar{R}} = \bar{\bar{P}}_y\bar{\bar{R}}_y(90° - \phi)\bar{\bar{R}}_z(\lambda^* - \lambda)\bar{\bar{R}}_y(\phi^* - 90°)\bar{\bar{P}}_y$$

$$\bar{\bar{R}} = \begin{bmatrix} \cos\beta\cos\alpha\cos\gamma - \sin\alpha\sin\gamma & \sin\beta\cos\alpha & \cos\beta\cos\alpha\sin\gamma + \sin\alpha\cos\gamma \\ -\sin\beta\cos\gamma & \cos\beta & -\sin\beta\sin\gamma \\ -\cos\beta\sin\alpha\cos\gamma - \cos\alpha\sin\gamma & -\sin\beta\sin\alpha & -\cos\beta\sin\alpha\sin\gamma + \cos\alpha\cos\gamma \end{bmatrix}$$

$$\bar{\bar{R}} = \begin{bmatrix} \cos\beta\sin\phi\sin\phi^* + \cos\phi\cos\phi^* & \sin\beta\sin\phi & -\cos\beta\sin\phi\cos\phi^* + \cos\phi\sin\phi^* \\ -\sin\beta\sin\phi^* & \cos\beta & \sin\beta\cos\phi^* \\ -\cos\beta\cos\phi\sin\phi^* + \sin\phi\cos\phi^* & -\sin\beta\cos\phi & \cos\beta\cos\phi\cos\phi^* + \sin\phi\sin\phi^* \end{bmatrix}$$

where

$$\begin{aligned} \alpha &= 90° - \phi \\ \beta &= \lambda^* - \lambda \\ \gamma &= \phi^* - 90° \\ \cos\alpha &= \sin\phi \\ \sin\alpha &= \cos\phi \\ \cos\gamma &= \sin\phi^* \\ \sin\gamma &= -\cos\phi^* \end{aligned}$$

Likewise, transformation for the gravity gradient tensor T is

$$\bar{\bar{T}} = \bar{\bar{R}}\bar{\bar{T}}^*\bar{\bar{R}}^T$$

5.6 Recommended reading

- Smith et al. (2001)
- Wild-Pfeiffer (2008)

5.7 References

Asgharzadeh, M. F., R. R. B. von Frese, H. R. Kim, T. E. Leftwich, and J. W. Kim (2007), Spherical prism gravity effects by Gauss-Legendre quadrature integration, Geophysical Journal International, 169(1), 1-11, doi:10.1111/j.1365-246X.2007.03214.x.

Grombein, T.; Seitz, K.; Heck, B. (2013), Optimized formulas for the gravitational field of a tesseroid, Journal of Geodesy, doi: 10.1007/s00190-013-0636-1

Nagy, D., G. Papp, and J. Benedek (2000), The gravitational potential and its derivatives for the prism, Journal of Geodesy, 74(7-8), 552-560, doi:10.1007/s001900000116.

Nagy, D., G. Papp, and J. Benedek (2002), Corrections to "The gravitational potential and its derivatives for the prism," Journal of Geodesy, 76(8), 475-475, doi:10.1007/s00190-002-0264-7.

Smith, D. A., D. S. Robertson, and D. G. Milbert (2001), Gravitational attraction of local crustal masses in spherical coordinates, Journal of Geodesy, 74(11-12), 783-795, doi:10.1007/s001900000142.

Wild-Pfeiffer, F. (2008), A comparison of different mass elements for use in gravity gradiometry, Journal of Geodesy, 82(10), 637-653, doi:10.1007/s00190-008-0219-8.

CHAPTER

SIX

USING TESSEROIDS

This is a tutorial about how to use the Tesseroids package. It is a work-in-progress but I have trie to be as complete as possible. If you find that anything is missing, or would like something explained in more detail, please submit a bug report (http://code.google.com/p/tesseroids/issues/list) (it's not that hard).

Any further questions and comments can be e-mail directly to me (leouieda [at] gmail [dot] com).

If you don't find what you're looking for here, the *cookbook* contains several example recipes of using Tesseroids.

6.1 A note about heights and units

In order to have a single convention, the word "height" means "height above the Earths surface" and are interpreted as positive up and negative down (i.e., oriented with the z axis of the Local coordinate system). Also, all input units are in SI and decimal degrees. Output of tesspot is in SI, tessgx, tessgy, and tessgz are in mGal, and the tensor components in Eotvos. All other output is also in SI and decimal degrees.

6.2 Getting help information

All programs accept the -h and –version flags. -h will print a help message describing the usage, input and output formats and options accepted. –version prints version and license information about the program.

Program tessdefaults prints the default values of constants used in the computations such as: mean Earth radius, pi, gravitational constant, etc.

6.3 Computing the gravitational effect of a tesseroid

The tesspot, tessgx, tessgy, tessgz, tessgxx, etc. programs calculate the combined effect of a list of tesseroids on given computation points. The computation points are passed via standard

input and do NOT have to be in a regular grid. This allows, for example, computation on points where data was measured. The values calculated are put in the last column of the input points and printed to standard output.

For example, if calculating gz on these points:

```
lon1 lat1 height1 value1 othervalue1
lon2 lat2 height2 value2 othervalue2
...
lonN latN heightN valueN othervalueN
```

the output would look something like:

```
lon1 lat1 height1 value1 othervalue1 gz1
lon2 lat2 height2 value2 othervalue2 gz2
...
lonN latN heightN valueN othervalueN gzN
```

The input model file should contain one tesseroid per line and have columns formatted as:

```
W E S N HEIGHT_OF_TOP HEIGHT_OF_BOTTOM DENSITY
```

HEIGHT_OF_TOP and HEIGHT_OF_BOTTOM are positive if the above the Earth's surface and negative if bellow.

Note: Remember that HEIGHT_OF_TOP > HEIGHT_OF_BOTTOM!

Use the command line option -h to view a list of all commands available.

Example:

Calculate the field of a tesseroid model having verbose printed and logged to file `gz.log` and GLQ order 3/3/3. The computation points are in `points.txt` and the output will be placed in `gz_data.txt`:

```
tessgz modelfile.txt -v -lgz.log -o3/3/3 < points.txt > gz_data.txt
```

6.4 The -a flag

The -a flag on tesspot, tessgx, tessgxx, etc., programs **disables** the automatic recursive dividing of tesseroids to maintain the GLQ accuracy. As a general rule, the tesseroid should be no bigger than a ratio times the distance from the computation point (program tessdefaults prints the value of the size ratios used). The programs automatically break the tesseroids when this criterion is breached. This means that the computations can be performed with the default GLQ order 2/2/2, which is much faster, and still maintain correctness.

Warning: It is strongly recommended that you don't use this flag unless you know what you are doing! It is also recommended that you keep 2/2/2 order always.

6.5 Verbose and logging to files

The -v flag enables printing of information messages to the default error stream (stderr). If omitted, only error messages will appear. The -l flag enables logging of information and error messages to a file.

6.6 Comments and provenance information

Comments can be inserted into input files by placing a "#" character at the start of a line. All comment lines are ignored. All programs pass on (print) the comment lines of the input to the output. All programs insert comments about the provenance of their results (where they came from) to their output. These include names of input files, version of program used, date, etc.

6.7 Generating regular grids

Included in the package is program tessgrd, which creates a regular grid of points and prints them to standard output.

Example

To generate a regular grid of 100 x 100 points, in the are -10/10/-10/10 degrees, at a height of 250 km:

```
tessgrd -r-10/10/-10/10 -b100/100 -z250e03 -v > points.txt
```

6.8 Automatic model generation

As of version 1.0, Tesseroids includes program tessmodgen for automatically generating a tesseroid model from a map of an interface. The interface can be any surface deviating from a reference level. For example, topography (a DEM) deviates from 0, a Moho map deviates from a mean crustal thickness, etc. This program takes as input a **REGULAR** grid with longitude, latitude and height values of the interface. Each tesseroid is generated with a grid point at the center of it's top face. The top and bottom faces of the tesseroid are defined as:

- Top = Interface and Bottom = Reference if the interface is above the reference
- Top = Reference and Bottom = Interface if the interface is bellow the reference

The density RHO of the tesseroids can be passed using the -d option. This will assign a density value of RHO, when the interface is above the reference, and a value of -RHO if the interface is bellow the reference. Alternatively, the density of each tesseroid can be passed as a forth column on the input grid. As with the -d option, if the interface is bellow the reference, the density value will be multiplied by -1! Also, an error will occur if both a forth column and the -d option are passed!

Example:

To generate a tesseroid model from a Digital Elevation Model (DEM) with 1 x 1 degree resolution using a density of 2670 km/m^3:

```
tessmodgen -s1/1 -d2670 -z0 -v < dem_file.txt > dem_tess_model.txt
```

6.9 Calculating the total mass of a model

The tessmass program can be used to compute the total mass of a given tesseroid model. If desired, a density range can be given and only tesseroids that fall within the given range will be used in the calculation.

Example:

To calculate the total mass of all tesseroids in `model.txt` with density between 0 and 1 g/cm^3:

```
tessmass -r0/1000 < model.txt
```

6.10 Computing the effect of rectangular prisms in Cartesian coordinates

Tesseroids 1.0 also introduced programs to calculate the gravitational effect of right rectangular prisms in Cartesian coordinates. This is done using the formula of Nagy et al. (2000). The programs are prismpot, prismgx, prismgy, prismgz, prismgxx, etc. Input and output for these programs is very similar to that of the tesspot, tessgx, etc., programs. Computation points are read from standard input and the prism model is read from a file. The model file should have the column format:

```
X1 X2 Y1 Y2 Z1 Z2 DENSITY
```

A note on the coordinate system:

As in Nagy et al. (2000), the coordinate system for the rectangular prism calculations has X axis pointing North, Y axis pointing East and Z axis pointing Down. This is important to note because it differs from the convention adopted for the tesseroids. In practice, this means that the g_{xz} and g_{yz} components of the prism and tesseroid will have different signs. This will not be such for the g_z component, though, because the convention for tesseroids is to have Z axis Down for this component only. See the Theoretical background section of the User Manual for more details on this.

6.11 Piping

Tesseroids was designed with the Unix philosophy in mind:

```
Write programs that do one thing and do it well.
Write programs to work together.
Write programs to handle text streams,
    because that is a universal interface.
```

Therefore, all tessg* programs and tessgrd can be piped together to calculate many components on a regular grid.

Example:

Given a tesseroids file `model.txt` as follows:

```
-1 1 -1 1 0 -10e03 -500
```

Running the following would calculate gz and gradient tensor of tesseroids in `model.txt` of a regular grid from -5W to 5E and -5S to 5N on 100x100 points at 250 km height. And the best of all is that it is done in parallel! If your system has multiple cores, this would mean a great increase in the computation time. All information regarding the computations will be logged to files gz.log, gxx.log, etc. These should include the information about how many times the tesseroid had to be split into smaller ones to guarantee GLQ accuracy:

```
tessgrd -r-5/5/-5/5 -b100/100 -z250e03 | \
tessgz model.txt -lgz.log | \
tessgxx model.txt -lgxx.log | \
tessgxy model.txt -lgxy.log | \
tessgxz model.txt -lgxz.log | \
tessgyy model.txt -lgyy.log | \
tessgyz model.txt -lgyz.log | \
tessgzz model.txt -lgzz.log > output.txt
```

CHAPTER

SEVEN

COOKBOOK

The following recipes can be found in the `cookbook` folder that comes with your *Tesseroids* download (along with shell and batch scripts and sample output):

7.1 Calculate the gravity gradient tensor from a DEM

This example demonstrates how to calculate the gravity gradient tensor (GGT) due to topographic masses using tesseroids.

To do that we need:

1. A DEM file with lon, lat, and height information;
2. Assign correct densities to continents and oceans (we'll be using a little Python for this);
3. Convert the DEM information into a tesseroid model;
4. Calculate the 6 components of the GGT;

The file `dem_brasil.sh` is a small shell script that executes all the above (we'll be looking at each step in more detail):

7.1.1 Why Python

Python is a modern programming language that is very easy to learn and extremely productive. We'll be using it to make our lives a bit easier during this example but it is by no means a necessity. The same thing could have been accomplished with Unix tools and the Generic Mapping Tools (http://www.soest.hawaii.edu/gmt) (GMT) or other plotting program.

If you have interest in learning Python we recommend the excelent video lectures in the Software Carpentry (http://software-carpentry.org) course. There you will also find lectures on various scientific programming topics. I strongly recommend taking this course to anyone who works with scientific computing.

7.1.2 The DEM file

For this example we'll use ETOPO1 (http://www.ngdc.noaa.gov/mgg/global/global.html) for our DEM. The file `dem.xyz` contains the DEM as a 10' grid. Longitude and latitude are in decimal degrees and heights are in meters. This is what the DEM file looks like (first few lines):

Notice that *Tesseroids* allows you to include comments in the files by starting a line with #. *This figure* shows the DEM ploted in pseudocolor. The red rectangle is the area in which we'll be calculating the GGT.

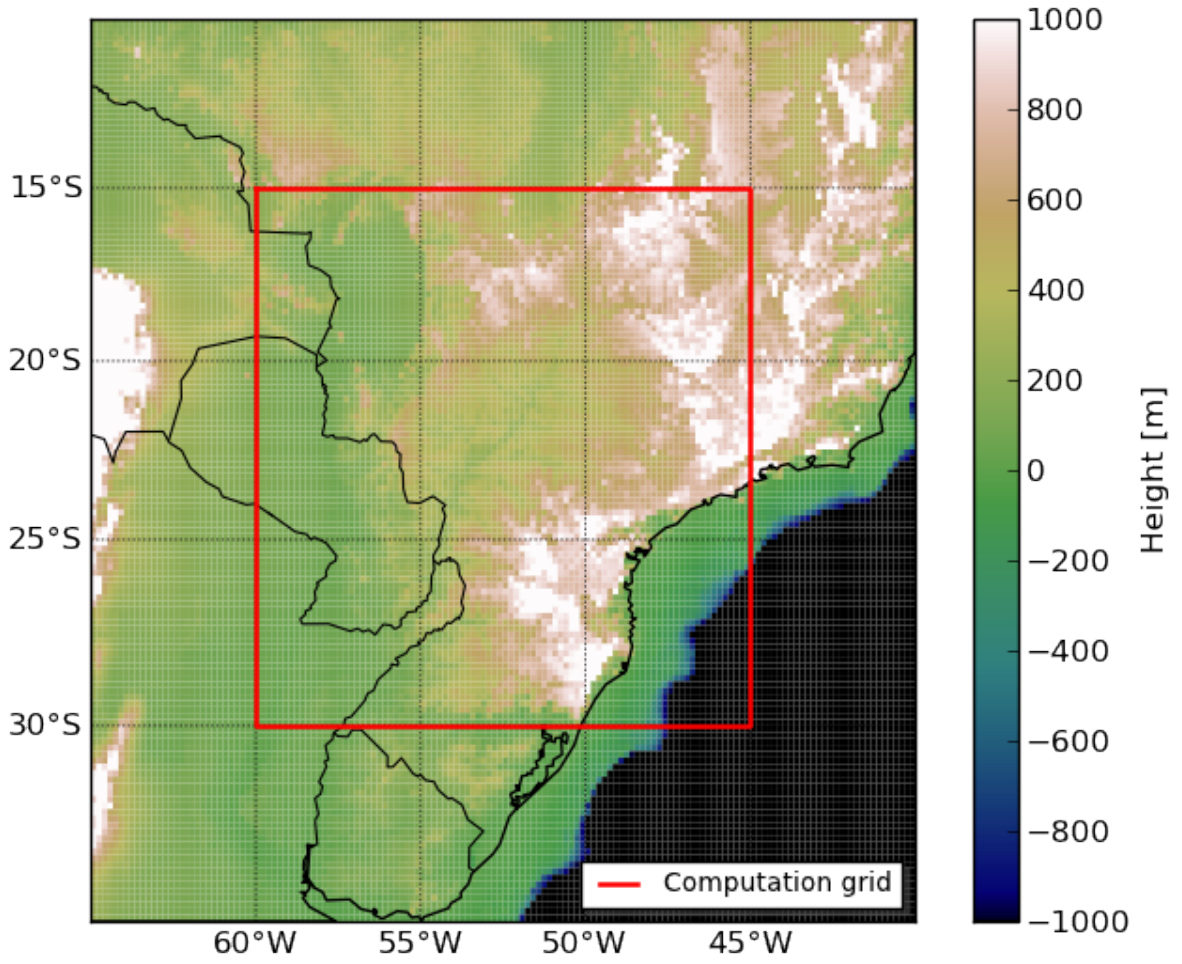


Figure 7.1: The ETOPO1 10' DEM of the Parana Basin, southern Brasil.

7.1.3 Assigning densities

Program tessmodgen allows us to provide the density value of each tesseroid through the DEM file. All we have to do is insert an extra column in the DEM file with the density values of the tesseroids that will be put on each point. This way we can have the continents with 2.67 g/cm3 and oceans with 1.67 g/cm3. Notice that the density assigned to the oceans is positive! This is because the DEM in the oceans will have heights bellow our reference (h = 0km) and tessmodgen will automatically invert the sign of the density values if a point is bellow the reference.

We will use the Python script `dem_density.py` to insert the density values into our DEM and save the result to `dem-dens.txt`:

If you don't know Python, you can easily do this step in any other language or even in Excel. This is what the `dem_density.py` script looks like:

The result is a DEM file with a forth column containing the density values (see *this figure*):

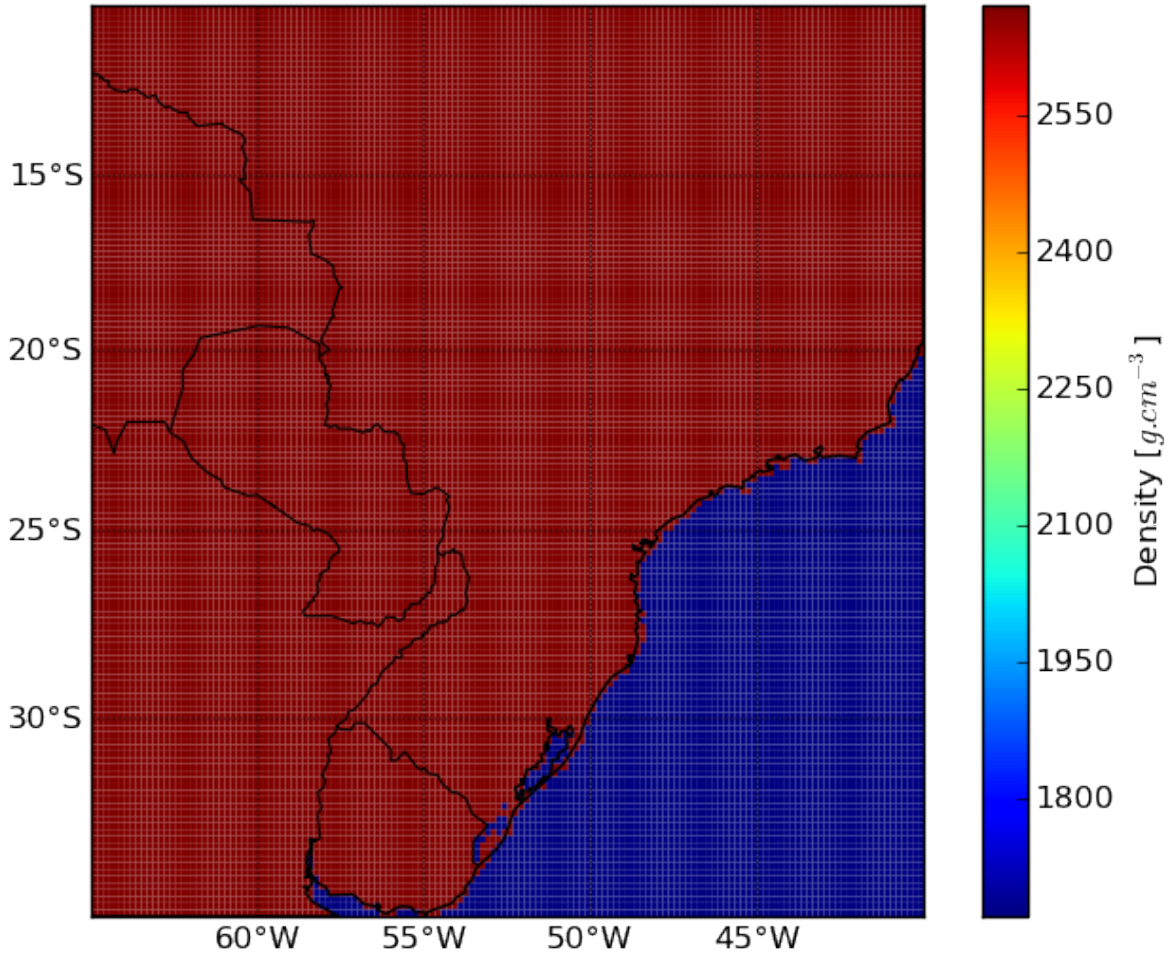


Figure 7.2: Density values. 2.67 g/cm3 in continents and 1.67 g/cm3 in the oceans.

7.1.4 Making the tesseroid model

Next, we'll use our new file `dem-dens.txt` and program tessmodgen to create a tesseroid model of the DEM:

tessmodgen places a tesseroid on each point of the DEM. The bottom of the tesseroid is placed on a reference level and the top on the DEM. If the height of the point is bellow the reference, the top and bottom will be inverted so that the tesseroid isn't upside-down. In this case, the density value of the point will also have its sign changed so that you get the right density values if modeling things like the Moho. For topographic masses, the reference surface is h = 0km (argument `-z`). The argument `-s` is used to specify the grid spacing (10') which will be used to

set the horizontal dimensions of the tesseroid. Since we didn't pass the `-d` argument with the density of the tesseroids, tessmodgen will expect a fourth column in the input with the density values.

The result is a tesseroid model file that should look somthing like this:

and for the points in the ocean (negative height):

7.1.5 Calculating the GGT

Tesseroids allows use of custom computation grids by reading the computation points from standard input. This way, if you have a file with lon, lat, and height coordinates and wish to calculate any gravitational field in those points, all you have to do is redirect stardard input to that file (using <). All tess* programs will calculate their respective field, append a column with the result to the input and print it to stdout. So you can pass grid files with more than three columns, as long as the first three correspond to lon, lat and height. This means that you can pipe the results from one tessg to the other and have an output file with many columns, each corresponding to a gravitational field. The main advantage of this approach is that, in most shell environments, the computation of pipes is done in parallel. So, if your system has more than one core, you can get parallel computation of GGT components with no extra effort.

For convience, we added the program tessgrd to the set of tools, which creates regular grids and print them to standard output. So if you don't want to compute on a custom grid (like us), you can simply pipe the output of tessgrd to the tess* programs:

The end result of this is file `dem-ggt.txt`, which will have 9 columns in total. The first three are the lon, lat and height coordinates generated by tessgrd. The next six will correspond to each component of the GGT calculated by tessgxx, tessgxy, etc., respectively. The resulting GGT is shown in *this figure*.

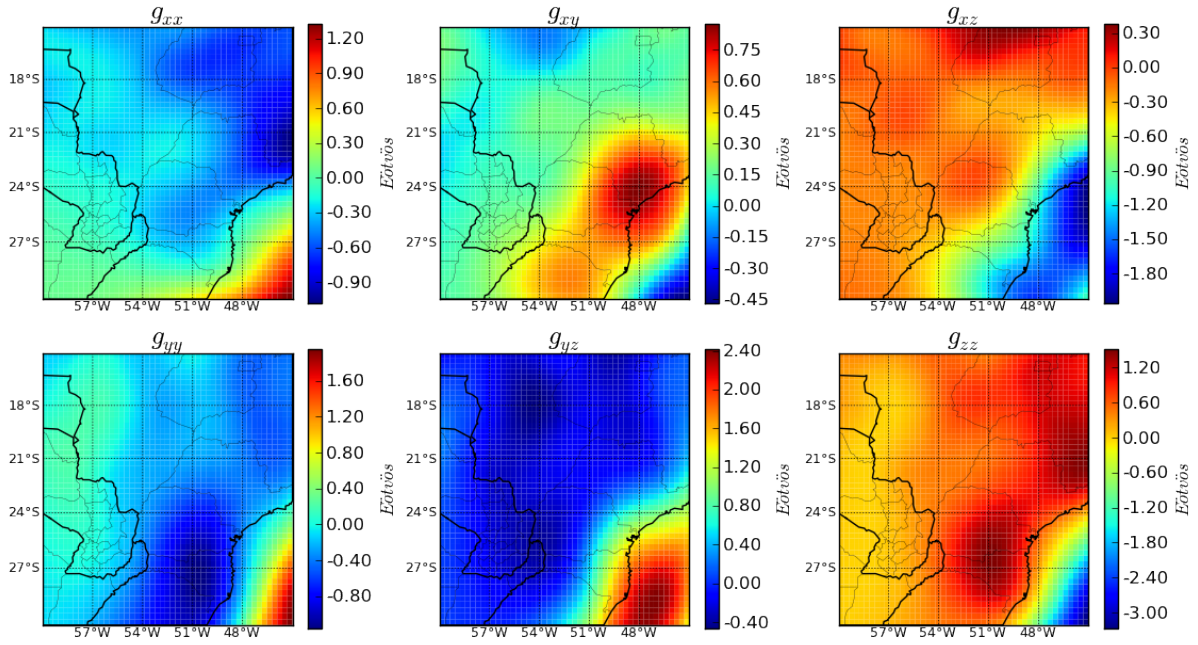


Figure 7.3: GGT of caused by the topographic masses.

7.1.6 Making the plots

The plots were generated using the powerfull Python library Matplotlib (http://matplotlib.sourceforge.net/index.html). The script `plots.py` is somewhat more complicated than `dem_density.py` and requires a bit of "Python Fu". The examples in the Matplotlib website should give some insight into how it works. To hanble the map projections, we used the Basemap toolkit (http://matplotlib.sourceforge.net/basemap/doc/html/) of Matplotlib.

7.2 Simple prism model in Cartesian coordinates

The `simple_prism.sh` script calculates the gravitational potential, gravitational attraction, and gravity gradient tensor due to a simple prism model in Cartesian coordinates:

```
#!/bin/bash

# Generate a regular grid, pipe it to all the computation programs,
# and write the result to output.txt

tessgrd -r0/20000/0/20000 -b50/50 -z1000 | \
prismpot model.txt | \
prismgx model.txt | prismgy model.txt | prismgz model.txt | \
prismgxx model.txt  | prismgxy model.txt  | \
prismgxz model.txt  | prismgyy model.txt  | \
prismgyz model.txt  | prismgzz model.txt > output.txt
```

The model file looks like this:

```
# Test prism model file
2000 5000 2000 15000 0 5000 1000
10000 18000 10000 18000 0 5000 -1000
```

The result should look like the *following* ("column" means the column of the output file).

7.3 Simple tesseroid model

The `simple_tess.sh` script calculates the gravitational potential, gravitational attraction, and gravity gradient tensor due to a simple tesseroid model:

```
#!/bin/bash

# Generate a regular grid, pipe it to all the computation programs,
# and write the result to output.txt

tessgrd -r-30/30/-30/30 -b50/50 -z250e03 | \
tesspot model.txt | \
tessgx model.txt | tessgy model.txt | tessgz model.txt | \
```

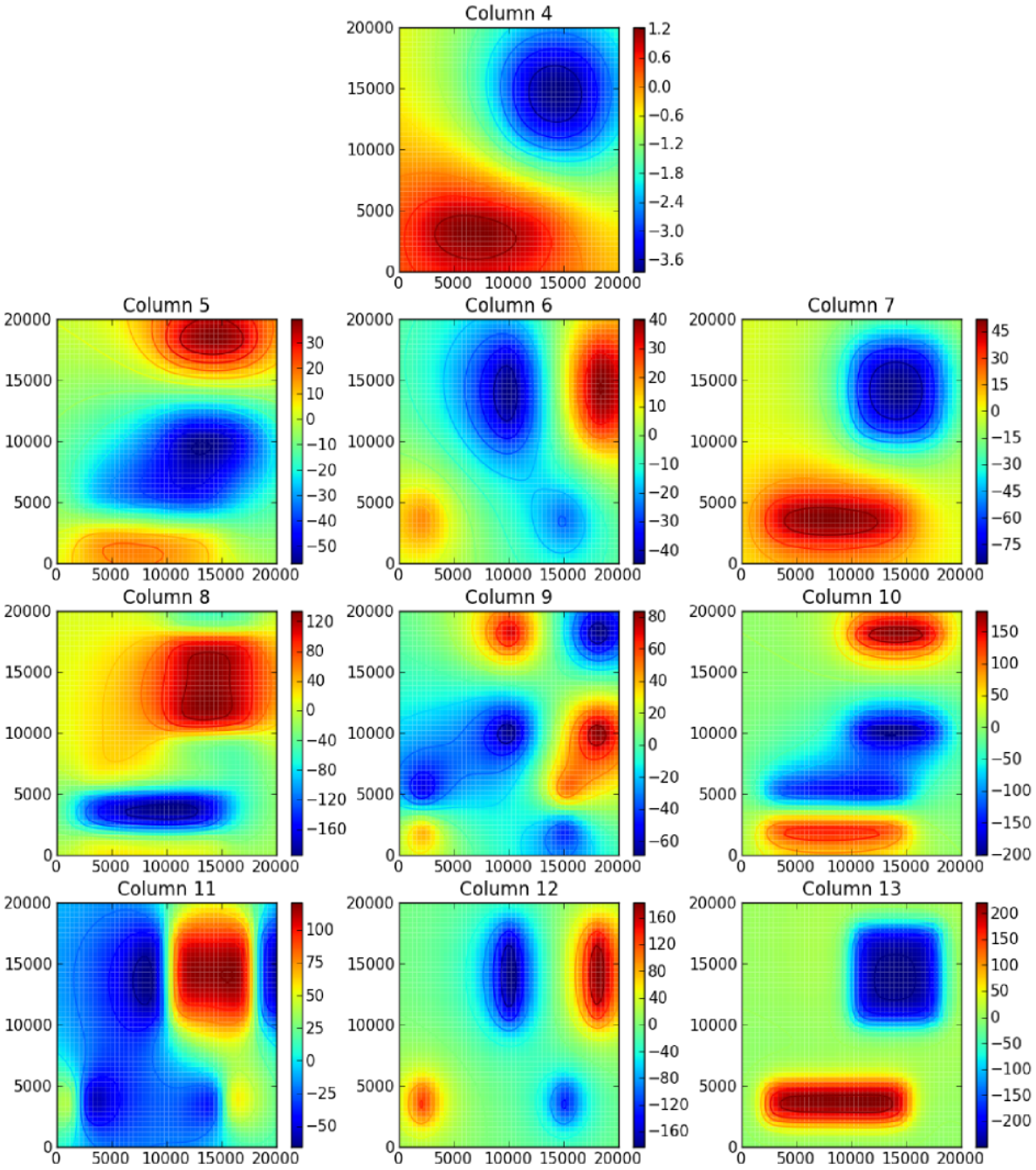


Figure 7.4: Plot of the columns of `output.txt` generated by `simple_prism.sh`. The x and y axis are longitude and latitude, respectively.

```
tessgxx model.txt  | tessgxy model.txt  | \
tessgxz model.txt  | tessgyy model.txt  | \
tessgyz model.txt  | tessgzz model.txt -v -llog.txt > output.txt
```

Option `-v` tells tessgzz to print information messages (to stderr). Option `-llog.txt` tells tessgzz to log the information plus debug messages to a file called `log.txt`.

The model file looks like this:

```
# Test tesseroid model file
-5 5 -10 10 0 -50000 200
-12 -16 -12 -16 0 -30000 -500
```

You will notice that this takes considerably more time to compute than a *simple 2 prism model*. This is because *Tesseroids* has to recursively divide each tesseroid into smaller tesseroids in order to maintain the accuracy of the *numerical integration*. If you have a look at `log.txt`, you'll notice that the 2 tesseroids were divided approximately 360,000 times in total. Don't worry, this only happened because the tesseroids are extremely large.

The result should look like the *following* ("column" means the column of the output file).

7.4 Convert a tesseroid model to prisms and calculate in spherical coordinates

The `tess2prism.sh` script converts a tesseroid model to prisms (using `tessmodgen`) and calculates the gravitational potential, gravitational attraction, and gravity gradient tensor in spherical coordinates:

```
#!/bin/bash

# Generate a prism model from a tesseroid model.
# Prisms will have the same mass as the tesseroids and
# associated spherical coordinates of the center of
# the top of the tesseroid.

tess2prism < tess-model.txt > prism-model.txt

# Generate a regular grid in spherical coordinates,
# pipe the grid to the computation programs,
# and dump the result on output.txt
# prismpots calculates the potential in spherical
# coordinates, prismgs calculates the full
# gravity vector, and prismggts calculates the full
# gravity gradient tensor.

tessgrd -r-160/0/-80/0 -b100/100 -z250e03 | \
prismpots prism-model.txt | \
prismgs prism-model.txt | \
prismggts prism-model.txt -v > output.txt
```

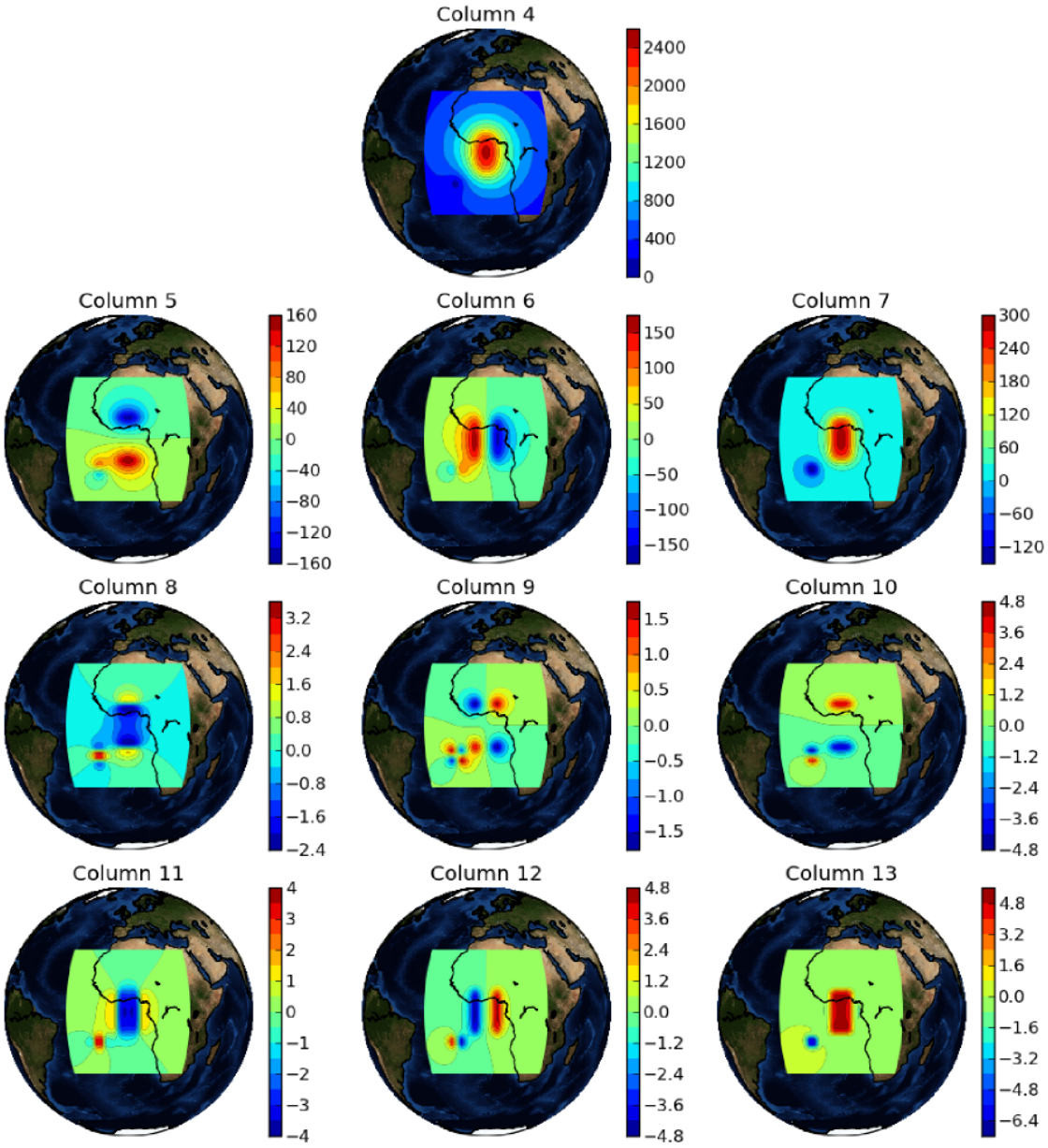


Figure 7.5: Plot of the columns of `output.txt` generated by `simple_tess.sh`. Orthographic projection (thanks to the Basemap (http://matplotlib.github.com/basemap/index.html) toolkit of matplotlib (http://matplotlib.sourceforge.net/index.html)).

The tesseroid model file looks like this:

```
# Test tesseroid model file
-77 -75 -41 -39 0 -50000 500
-79 -77 -41 -39 0 -50000 500
-81 -79 -41 -39 0 -50000 500
-83 -81 -41 -39 0 -50000 500
-85 -83 -41 -39 0 -50000 500
```

and the converted prism model looks like this:

```
# Prisms converted from tesseroid model with tess2prism 1.1dev
#   local time: Wed May 16 14:34:47 2012
#   tesseroids file: stdin
#   conversion type: equal mass|spherical coordinates
#   format: dx dy dz density lon lat r
# Test tesseroid model file
221766.31696055 169882.854778591 50000 499.977196258595 -76 -40 6378137
221766.31696055 169882.854778591 50000 499.977196258595 -78 -40 6378137
221766.31696055 169882.854778591 50000 499.977196258595 -80 -40 6378137
221766.31696055 169882.854778591 50000 499.977196258595 -82 -40 6378137
221766.31696055 169882.854778591 50000 499.977196258595 -84 -40 6378137
```

Note that the density of prisms is altered. This is so that the tesseroid and corresponding prism have the same mass.

The result should look like the *following* ("column" means the column of the output file).

7.5 Convert a tesseroid model to prisms and calculate in Cartesian coordinates

The `tess2prism_flatten.sh` script converts a tesseroid model to prisms (using the `--flatten` flag in `tessmodgen`) and calculates the gravitational potential, gravitational attraction, and gravity gradient tensor in Cartesian coordinates:

```
#!/bin/bash

# Generate a prism model from a tesseroid model by
# flattening the tesseroids (1 degree = 111.11 km).
# This way the converted prisms can be used
# with the prism* programs in Cartesian coordinates.

tess2prism --flatten < tess-model.txt > prism-model.txt

# Generate a regular grid in Cartesian coordinates,
# pipe the grid to the computation programs,
# and dump the result on output.txt

tessgrd -r-3e06/3e06/-3e06/3e06 -b50/50 -z250e03 | \
prismpot prism-model.txt | \
```

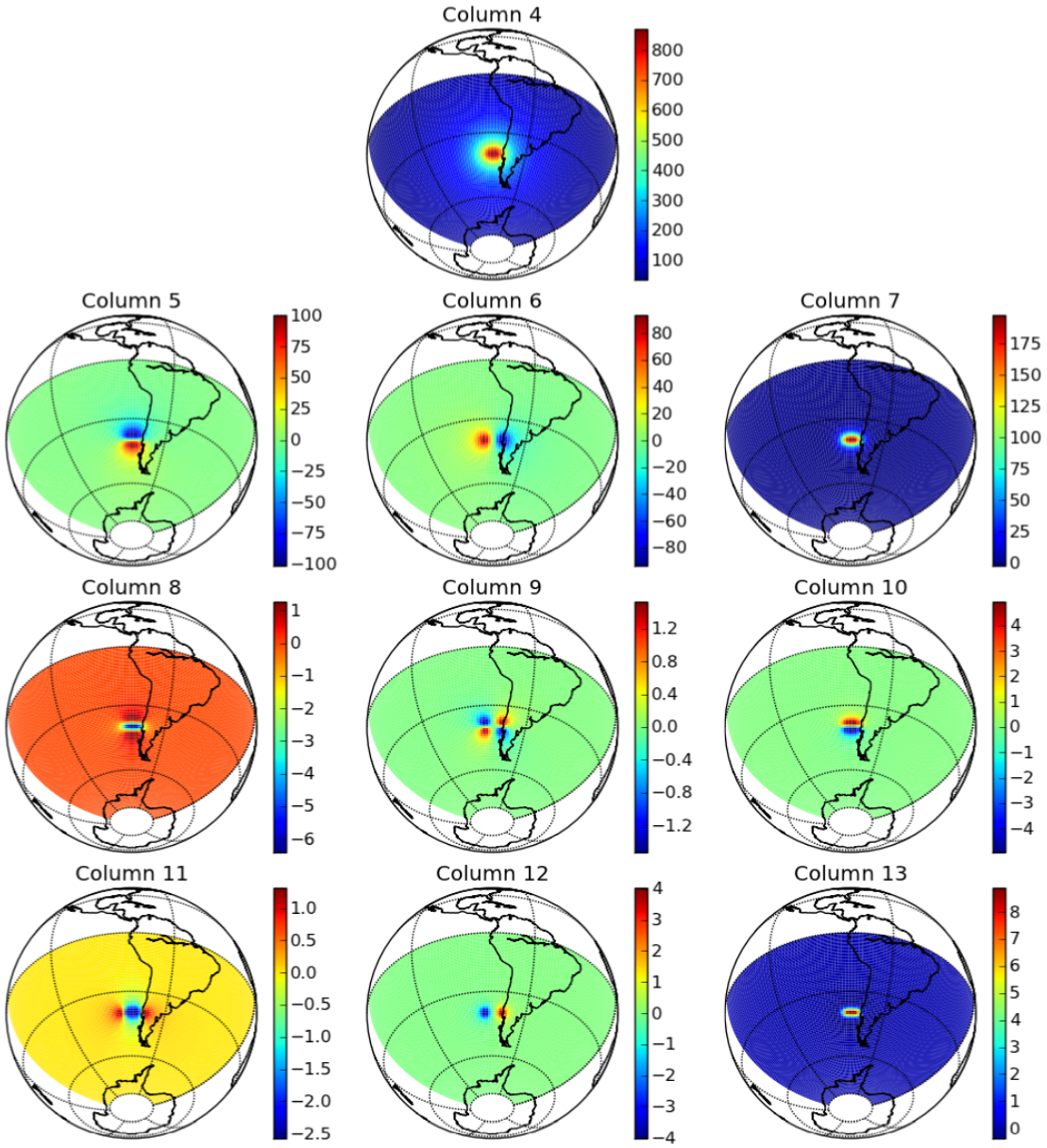


Figure 7.6: Plot of the columns of `output.txt` generated by `tess2prism.sh`. Orthographic projection (thanks to the Basemap (http://matplotlib.github.com/basemap/index.html) toolkit of matplotlib (http://matplotlib.sourceforge.net/index.html)).

```
prismgx prism-model.txt | \
prismgy prism-model.txt | \
prismgz prism-model.txt | \
prismgxx prism-model.txt  | prismgxy prism-model.txt  | \
prismgxz prism-model.txt  | prismgyy prism-model.txt  | \
prismgyz prism-model.txt  | prismgzz prism-model.txt > output.txt
```

The tesseroid model file looks like this:

```
# Test tesseroid model file
10 15 10 15 0 -30000 500
-15 -10 -10 10 0 -50000 200
-15 5 -16 -10 0 -30000 -300
```

and the converted prism model looks like this:

```
# Prisms converted from tesseroid model with tess2prism 1.1dev
#   local time: Tue May  8 14:55:02 2012
#   tesseroids file: stdin
#   conversion type: flatten
#   format: x1 x2 y1 y2 z1 z2 density
# Test tesseroid model file
1111100 1666650 1111100 1666650 0 30000 487.534658568521
-1111100 1111100 -1666650 -1111100 0 50000 198.175508383774
-1777760 -1111100 -1666650 555550 0 30000 -291.9029748328
```

Note that the density of prisms is altered. This is so that the tesseroid and corresponding prism have the same mass.

The result should look like the *following* ("column" means the column of the output file).

7.6 Using tesslayers to make a tesseroid model of a stack of layers

The `tesslayers.sh` script converts grids that define a stack of layers into a tesseroid model. It then calculates the gravitational attraction and gravity gradient tensor due to the tesseroid model:

```
#!/bin/bash

# Convert the layer grids in layers.txt to tesseroids.
# The grid spacing passed to -s is used as the size of the tesseroids,
# so be careful!
tesslayers -s0.5/0.5 -v < layers.txt > tessmodel.txt

# Now calculate the gz and tensor effect of this model at 100km height
tessgrd -r-8/8/32/48 -b50/50 -z100000 | \
tessgz tessmodel.txt | \
tessgxx tessmodel.txt | tessgxy tessmodel.txt | \
```

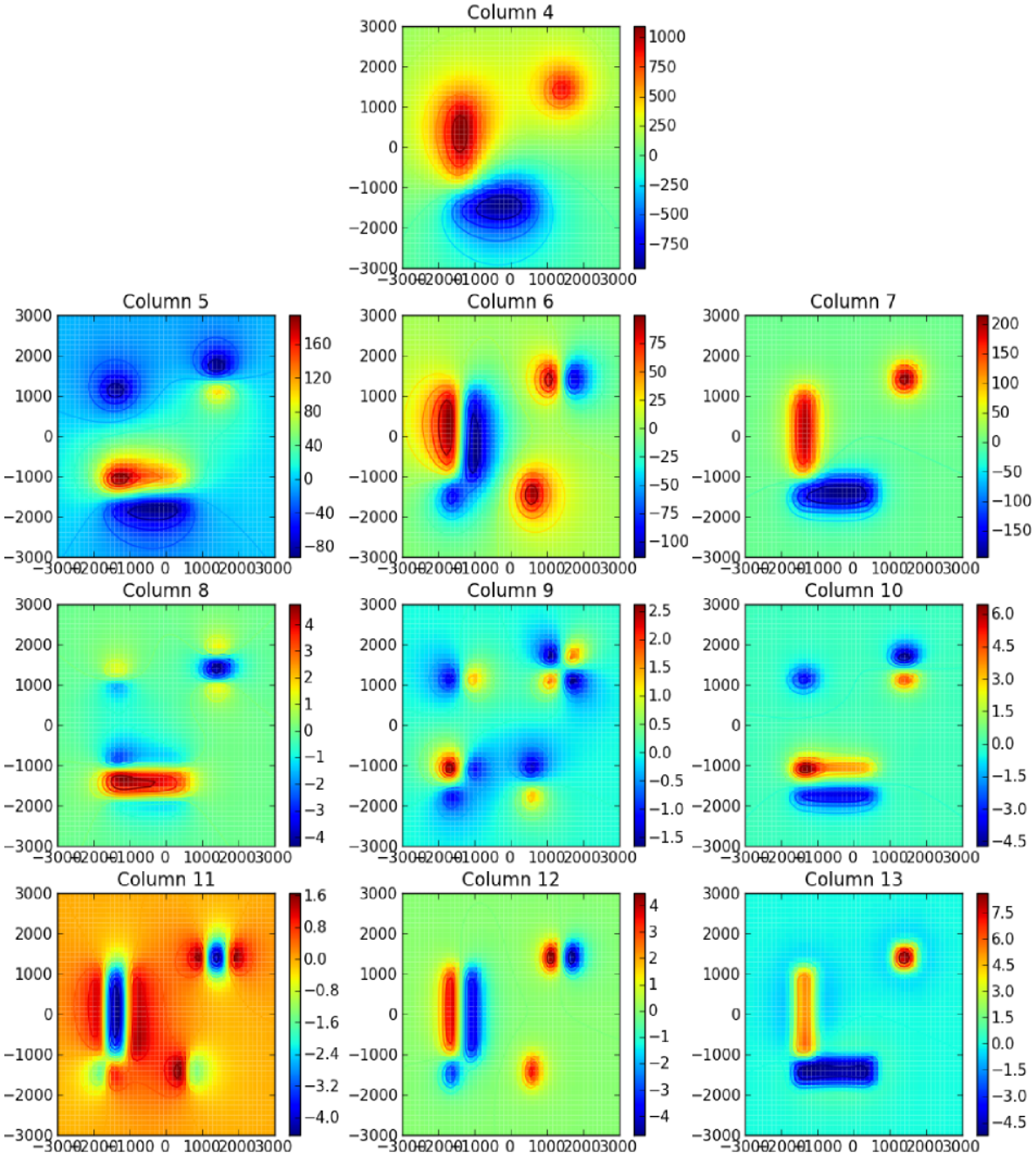


Figure 7.7: Plot of the columns of `output.txt` generated by `tess2prism_flatten.sh`. The x and y axis are West-East and South-North, respectively, in kilometers.

```
tessgxz tessmodel.txt | tessgyy tessmodel.txt | \
tessgyz tessmodel.txt | tessgzz tessmodel.txt -v > output.txt
```

The input file `layers.txt` contains the information about the stack of layers. It is basicaly regular grids in xyz format (i.e., in columns). The first 2 columns in the file are the longitude and latitude of the grid points. Then comes a column with the height of the first layer. This is the height (with respect to mean Earth radius) of the top of stack of layers. Then comes the thickness and density of each layer. Our layer file looks like this:

...

This is a synthetic layer model generated from two gaussian functions. *This* is what the topography (height column) and the thickness of the sediments look like:

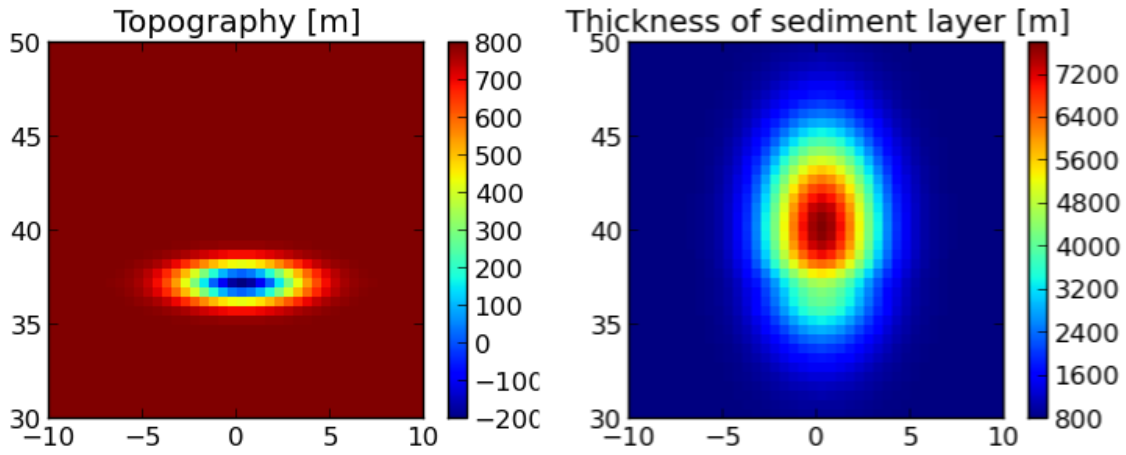


Figure 7.8: Plot of the third and forth columns of `layers.txt`. The x and y axis are longitude and latitude, respectively.

The model file generated looks like this:

...

The result should look like the *following* ("column" means the column of the output file).

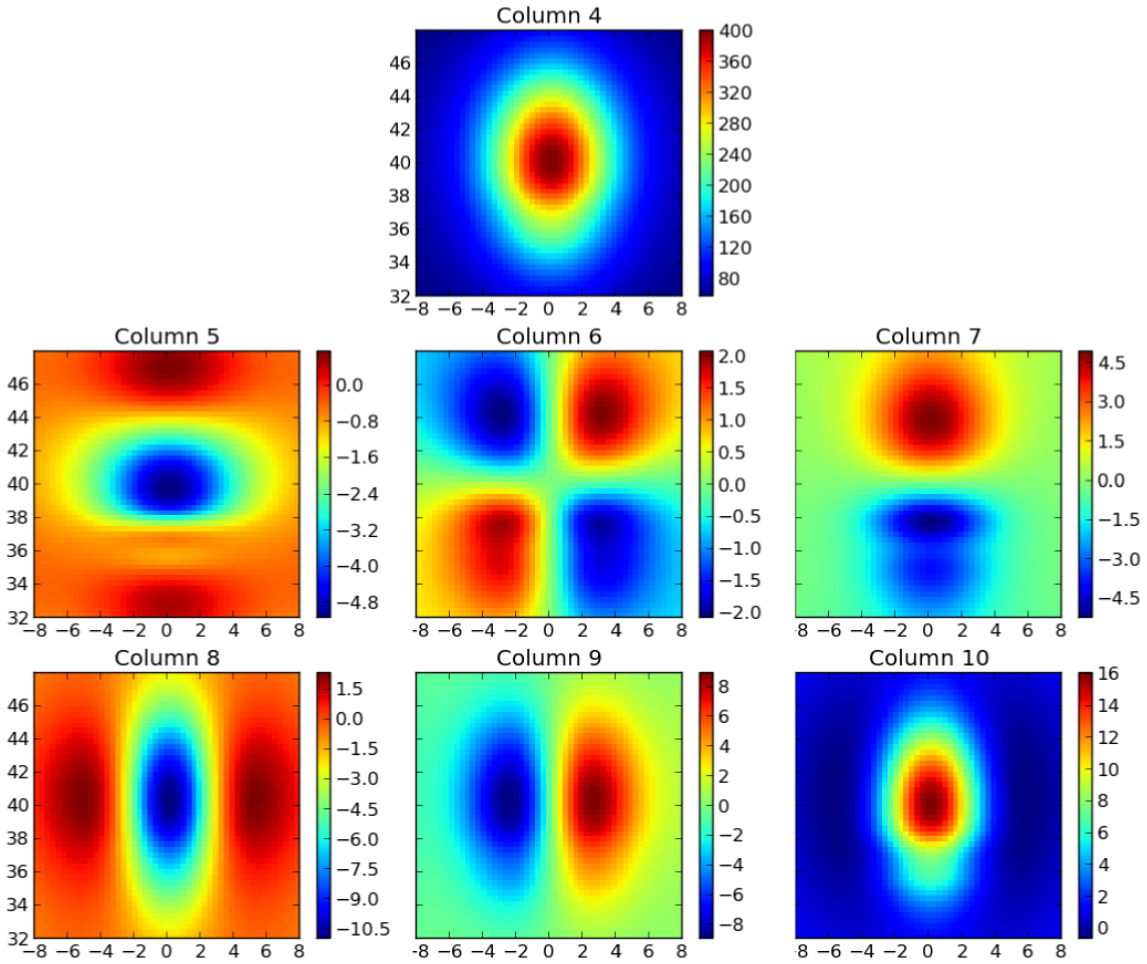


Figure 7.9: Plot of the columns of `output.txt` generated by `tesslayers.sh`. The x and y axis are longitude and latitude, respectively.